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Civil Services Main Examination Previous Solved Papers : Mechanical Engg. (Paper-I)

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Preface

Civil Service is considered as the most prestigious job in India and it has become a preferred destination by all engineers. In order to reach this estimable position every aspirant has to take arduous journey of Civil Services Examination (CSE). Focused approach and strong determination are the pre-requisites for this journey. Besides this, a good book also comes in the list of essential commodity of this odyssey.



I feel extremely glad to launch the revised edition of such a book which will not only make CSE plain sailing, but also with 100% clarity in concepts.

MADE EASY team has prepared this book with utmost care and thorough study of all previous years papers of CSE. The book aims to provide complete solution to all previous years questions with accuracy.

On doing a detailed analysis of previous years CSE question papers, it came to light that a good percentage of questions have been asked in Engineering Services, Indian Forest Service and State Services exams. Hence, this book is a one stop shop for all CSE, ESE, IFS and other competitive exam aspirants.

I would like to acknowledge efforts of entire MADE EASY team who worked day and night to solve previous years papers in a limited time frame and I hope this book will prove to be an essential tool to succeed in competitive exams and my desire to serve student fraternity by providing best study material and quality guidance will get accomplished.

With Best Wishes

B. Singh (Ex. IES)

CMD, MADE EASY Group

Previous Years Solved Papers of
Civil Services Main Examination

Mechanical Engineering : Paper-I

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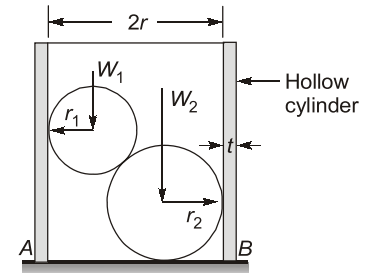
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Engineering Mechanics

1. Equations of Equilibrium and Statics

Q.1 A smooth hollow cylinder of radius r open at both ends rests on a smooth horizontal plane. Two smooth spheres of weights W_1 and W_2 and radii r_1 and r_2 , respectively are placed inside the cylinder, with the larger sphere (radius r_2) resting on the horizontal plane as shown in figure.

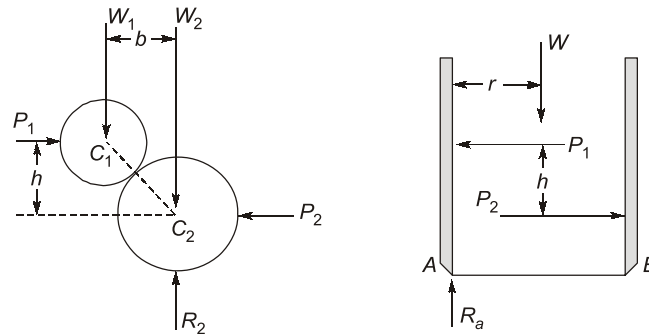
Determine the minimum weight W of the cylinder that will prevent the cylinder from tipping over.



[CSE (Mains) 2009 : 12 Marks]

Solution:

Let us consider first a free-body for the two sphere, assuming that they are joined together at their point of contact as shown in figure. By virtue of the assumptions of smooth surfaces, we conclude that the reactive forces P_1 and P_2 exerted on the spheres by the walls of the cylinder are horizontal forces as shown and likewise that the reaction R_2 on the bottom of the lower sphere is a vertical force.



Thus the two spheres are in equilibrium under the action of the five coplanar forces shown in figure.

By equilibrium equation in horizontal axis,

$$P_1 - P_2 = 0 \quad \dots(i)$$

Taking moment about point C_2 ,

$$W_1 b - P_1 h = 0 \quad \dots(ii)$$

From equations (i) and (ii), $P_1 = P_2 = \frac{W_1 b}{h}$

Under equilibrium condition at the verge of tipping, there will be no contact at point B . Force P_1 and P_2

constitute a couple, $M = P_1 \times h = \frac{W_1 b}{h} \times h = W_1 b$

Taking moment about point A , $Wr = W_1 b$

From which, $W = W_1 \frac{b}{r}$

$$b = 2r - r_1 - r_2$$

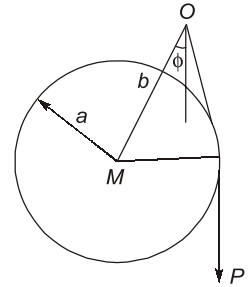
Expressions becomes,

$$W = W_1 \frac{(2r - r_1 - r_2)}{r}$$

$$\text{Minimum weight of cylinder, } W = W_1 \left(2 - \frac{r_1 + r_2}{r} \right)$$

Q.2 A homogeneous ball of weight Q and radius a as well as a weight P are suspended by cords from a point O as shown in figure. The distance OM is b . Find the inclination ϕ of OM with the vertical when the system is in equilibrium.

[CSE (Mains) 2009 : 8 Marks]



Solution:

Given: Weight of ball = Q , Radius = a , weight = P , $OM = b$.

Let inclination of OM with vertical when the system is in equilibrium be (ϕ) .

$$OM = b$$

$$MN = b \sin \phi \text{ and } ON = b \cos \phi$$

$$MX = a$$

$$NX = MX - MN = a - b \sin \phi$$

Taking moments about point O .

$$P \times (NX) = Q \times (MM)$$

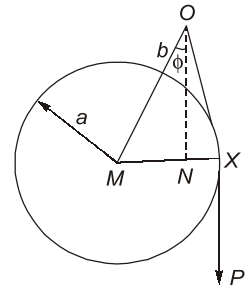
$$P(a - b \sin \phi) = Q(b \sin \phi)$$

$$Pa - Pb \sin \phi = Qb \sin \phi$$

$$Pa = Pb \sin \phi + Qb \sin \phi = (Pb + Qb) \sin \phi$$

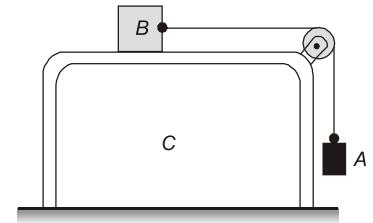
$$\sin \phi = \left(\frac{Pa}{Pb + Qb} \right)$$

$$\phi = \sin^{-1} \left(\frac{Pa}{Pb + Qb} \right) = \sin^{-1} \left(\frac{Pa}{b(P + Q)} \right)$$



Q.3 A body A of weight 10 kN is connected to another body B of weight 50 kN , resting on a smooth table of weight 200 kN through an inextensible thread, passing over a freely rotating pulley mounted on a corner of the table. Find the vertical component of the reaction of the ground on the table when the bodies A and B are in motion. Does the reaction change with time? The system is shown in figure.

[CSE (Mains) 2009 : 12 Marks]



Solution:

Given data:

Weight of A ,

$$W_A = 10 \text{ kN} = 10000 \text{ N}$$

Weight of B ,

$$W_B = 50 \text{ kN} = 5000 \text{ N}$$

Weight of table,

$$W_C = 200 \text{ kN} = 200 \times 10^3 \text{ N}$$

$$m_A = \frac{W_A}{g} = \frac{10000}{9.81} = \frac{10 \times 10^3}{9.81} = 1019.36 \text{ kg}$$

$$m_B = \frac{W_B}{g} = \frac{5000}{9.81} = \frac{50 \times 10^3}{9.81} = 5096.84 \text{ kg}$$

$$m_C = \frac{W_C}{g} = \frac{200 \times 10^3}{9.81} = 20387.36 \text{ kg}$$

Let T be the tension in string and N be the normal force, and acceleration of the system is a .

For body 'B'

$$T = m_B a = (5096.84) a \quad \dots (i)$$

$$N_B = W_B = m_B g \quad \dots (ii)$$

For body 'A'

$$m_A g - T = m_A a$$

$$\Rightarrow (1019.36)9.81 - T = (1019.36)a \quad \dots (iii)$$

From equations (i) and (ii),

$$(1019.36)(9.81) - (5096.84)a = (1019.36)a$$

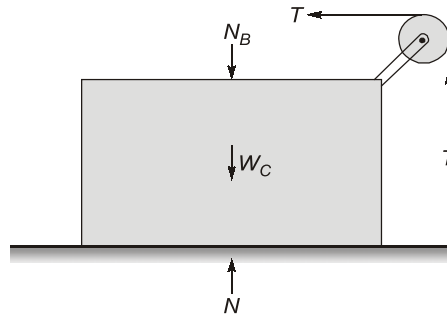
$$a = \frac{1019.36 \times 9.81}{6116.19} = 1.634 \text{ m/ sec}^2$$

$$T = m_B a = (5096.84) (1.634) = 8328.23 \text{ N} = 8.328 \text{ kN}$$

Considering forces on table

Let vertical component of the reaction of ground on table is 'N'.

Considering equilibrium of forces in vertical direction on block

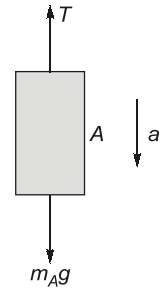
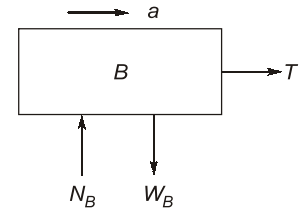


$$N - N_B - W_C - T = 0$$

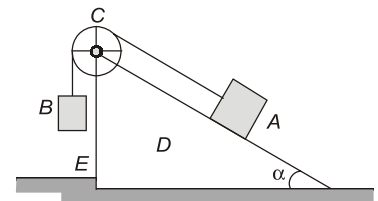
$$N = N_B + W_C + T = m_B g + W_C + T = 50 + 200 + 8.328 = 258.328 \text{ kN}$$

Vertical component of the reaction of ground on table $N = 258.328 \text{ kN}$.

The reaction is constant and will not change with time.



Q.4 A body A weighing P_1 descends down an inclined plane D which makes an angle α with the horizontal and pulls a load B that weighs P_2 by means of a weightless and inextensible string passing over a pulley C as shown in figure. Determine the horizontal component of the pressure with which the inclined plane D acts on the floor rib E .
[CSE (Mains) 2010 : 20 Marks]



Solution:

Given: Weight of body $A = P_1$
Weight of body $B = P_2$

For body A

$$P_1 \sin \alpha - T = \left(\frac{P_1}{g}\right)a \quad \dots (i)$$

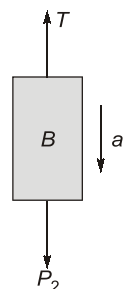
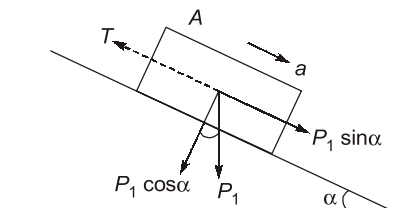
$$N = P_1 \cos \alpha \quad \dots (ii)$$

For body B ,

$$T - P_2 = \left(\frac{P_2}{g}\right)a \quad \dots (iii)$$

From equations (i) and (ii), we get

$$P_1 \sin \alpha - \left(P_2 + \left\{\frac{P_2}{g}\right\}a\right) = \left(\frac{P_1}{g}\right)a$$



$$a = \frac{P_1 \sin \alpha - P_2}{(P_1 + P_2) / g}$$

Putting the value of 'a' in equation (iii), we get

$$\begin{aligned} T - P_2 &= \left(\frac{P_2}{g}\right) \times \left(\frac{P_1 \sin \alpha - P_2}{(P_1 + P_2) / g}\right) \\ T &= P_2 + \frac{P_2(P_1 \sin \alpha - P_2)}{(P_1 + P_2)} = \frac{P_1 P_2 + P_2^2 + P_1 P_2 \sin \alpha - P_2^2}{P_1 + P_2} \\ &= \frac{P_1 P_2 + P_1 P_2 \sin \alpha}{P_1 + P_2} = \frac{P_1 P_2 (1 + \sin \alpha)}{P_1 + P_2} \end{aligned}$$

Forces on inclined plane, D

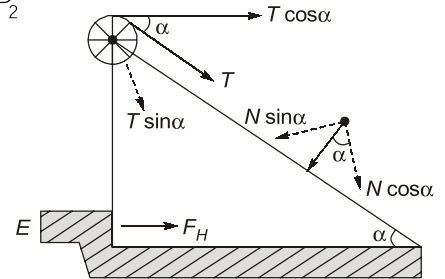
Let the horizontal component of the pressure with which the inclined plane D acts on the floor rib E be F_H

$$F_H - N \sin \alpha + T \cos \alpha = 0 \quad \dots (iv)$$

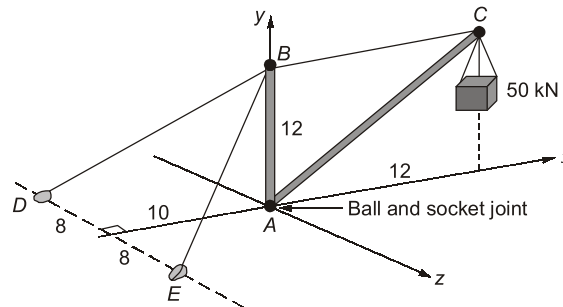
Putting the value of 'N' from equation (ii) and value of T in equation (iv), we get

$$F_H - (P_1 \cos \alpha) \sin \alpha + \frac{P_1 P_2 (1 + \sin \alpha)}{P_1 + P_2} \cos \alpha = 0$$

$$F_H = P_1 \cos \alpha \times \sin \alpha - \frac{P_1 P_2 (1 + \sin \alpha) \cos \alpha}{P_1 + P_2}$$



- Q.5 The crane shown in figure below is supported by cables BD and BE . The distances marked are in m. What are the tensions in the cables BC , BD and BE , when BC is parallel to the x -axis?



[CSE (Mains) 2010 : 8 Marks]

Solution:

Coordinate of different points are :-

$$D(-10, 0, -8), E(-10, 0, 8), B(0, 12, 0), C(12, 12, 0)$$

At point C :

$$\tan \theta = \frac{12}{12} \Rightarrow \theta = 45^\circ$$

$$T_{AC} \sin 45^\circ = 50 \text{ kN}$$

$$T_{AC} = 50\sqrt{2} \text{ kN}$$

$$T_{BC} = T_{AC} \cos 45^\circ = 50 \text{ kN}$$

By symmetry at point B ,

$$T_{BD} = T_{BE}$$

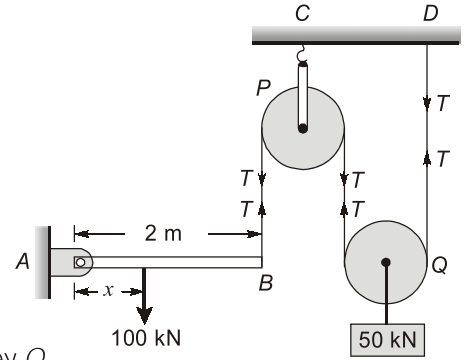
$$T_{BC} = 2 \times T_{BE} \times \frac{\sqrt{164}}{\sqrt{164+144}} \times \frac{10}{\sqrt{164}}$$

$$\Rightarrow 50 = 1.1396 T_{BE}$$

$$\Rightarrow T_{BE} = 43.874 \text{ kN} = T_{BD}$$

Q.6 A beam AB of length 2 m, hinged at A and supported at B by a cord which passes over two frictionless pulleys (P, Q), carries a 50 kN load as shown in figure. Determine the distance x , where 100 kN load is located on the beam, if the beam is to remain in equilibrium in horizontal position. Also determine the reaction at the hinged end.

[CSE (Mains) 2017 : 10 Marks]



Solution:

Let us consider free body diagram of beam AB and frictionless pulley Q .

For pulley Q : $\Sigma F_y = 0$ (force equilibrium)

$$2T = 50 \text{ or } T = 25 \text{ kN}$$

For beam AB : $\Sigma M_A = 0$ (moment equilibrium)

$$T \times 2 = 100 \times x \Rightarrow x = \frac{2T}{100} = \frac{2 \times 25}{100} = 0.5 \text{ m}$$

Vertical Force Equilibrium:

$$\Sigma F_y = 0$$

$$A_y + T = 100 \Rightarrow$$

$$A_y = 100 - 25 = 75 \text{ kN}$$

$$\Sigma F_x = 0$$

Horizontal Force Equilibrium:

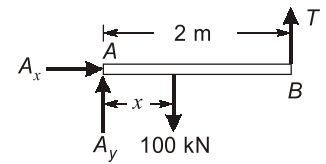
$$\text{i.e., } A_x = 0$$

The answers are :

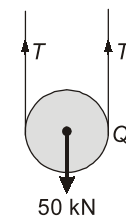
Location of 100 kN load i.e. $x = 0.5 \text{ m}$

Vertical reaction at support $A = 75 \text{ kN}$ (upward)

Horizontal reaction at support $A = 0 \text{ kN}$



FBD of beam AB



FBD of pulley Q

Q.7 A cord ACB , 5 m long is attached at points A and B to the vertical walls, 3 m apart (Figure 1). A pulley of negligible radius carries a suspended load of 200 N and is free to roll without friction along the cord. Determine the position of equilibrium as defined by the distance X , that the pulley will assume and also the tensile force in the cord.

[CSE (Mains) 2018 : 10 Marks]

Solution:

Given data:

Length of cord $ACB = 5 \text{ m}$

Length of $DB = 5 \text{ m}$

Considering ΔDCP and ΔDBE

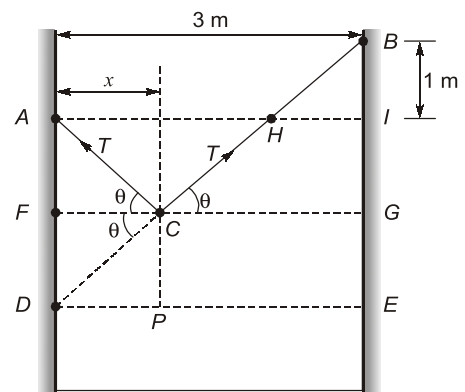
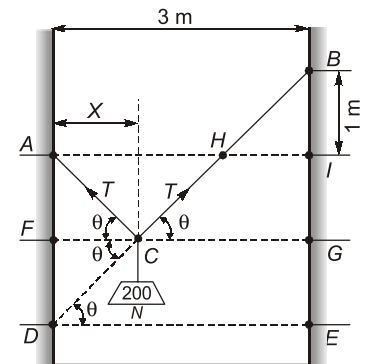
$$\frac{CP}{BE} = \frac{DP}{DE} = \frac{DC}{DB} \quad \dots (i)$$

$$DB = 5 \text{ m}$$

$$DE = 3 \text{ m}$$

$$BE = \sqrt{(DB)^2 - (DE)^2} = \sqrt{(5)^2 - (3)^2} = 4 \text{ m}$$

$$AF = FD = IG = GE = \frac{4-1}{2} = 1.5 \text{ m}$$



$$PC = FD = 1.5 \text{ m}$$

From equation (i),

$$\frac{DP}{DE} = \frac{PC}{BE}$$

⇒

$$\frac{x}{3} = \frac{1.5}{4}$$

$$x = \frac{3 \times 1.5}{4} = 1.125 \text{ m}$$

For the equilibrium,

$$2T \sin \theta = 200$$

and

$$\tan \theta = \frac{AF}{FC} = \frac{1.5}{1.125} = \frac{4}{3}$$

$$\sin \theta = \frac{4}{5}$$

Putting the value of $\sin \theta$, in equation (ii),

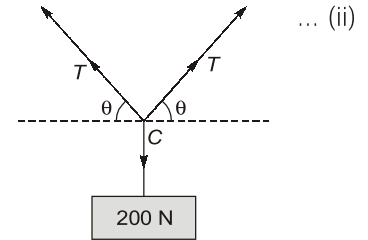
$$2T \left(\frac{4}{5} \right) = 200$$

⇒

$$T = \frac{200 \times 5}{8} = 125 \text{ N}$$

Position of equilibrium that the pulley will assume, $x = 1.125 \text{ m}$

Tensile force, $T = 125 \text{ N}$



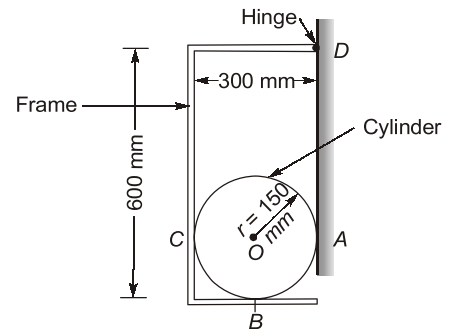
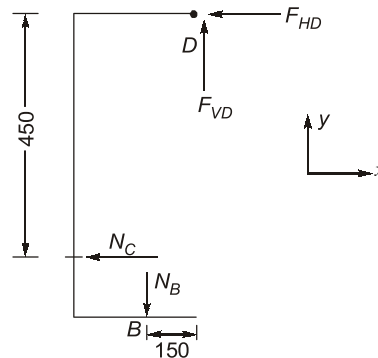
(Free body diagram at pulley)

Q.8 A 600N cylinder is supported by the frame BCD as shown in the figure. The frame is hinged at D . Determine the reactions at A , B , C and D .

[CSE (Mains) 2019 : 10 Marks]

Solution:

Drawing the FBD of the frame.



Since the frame is stationary

∴ Net moment of forces about D is zero.

∴

$$\Sigma M_D = 0$$

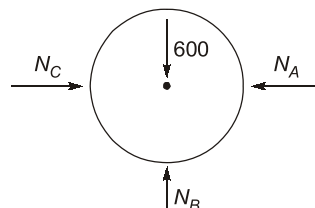
⇒

$$N_C \times 450 = N_B \times 150$$

⇒

$$N_B = 3N_C$$

Drawing FBD of the cylinder



$$\begin{array}{l|l} \Sigma F_x = 0 & \Sigma F_y = 0 \\ \hline F_{HD} = -N_C & F_{VD} = N_B \end{array} \quad \dots(i)$$

Net force in horizontal direction is zero.

$$\therefore N_A = N_C \quad \dots (ii)$$

Also, net forces in vertical direction is zero.

$$N_B = 600 \text{ N} \quad \dots (iii)$$

from equations (i), (ii) and (iii), $N_A = N_C = \frac{N_B}{3} = 200 \text{ N}$

$$N_B = 600 \text{ N}$$

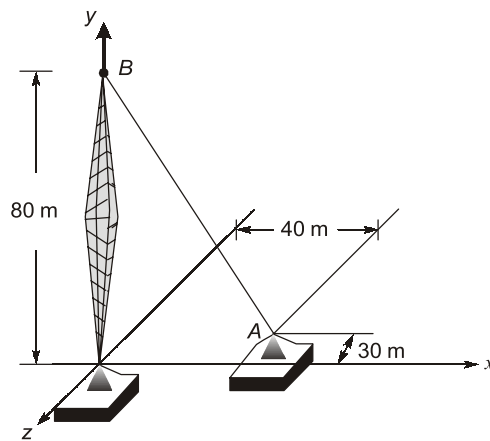
Reaction at point D,

$$F_{HD} = -N_C = -200 \text{ N}$$

$$F_{VD} = N_B = 600 \text{ N}$$

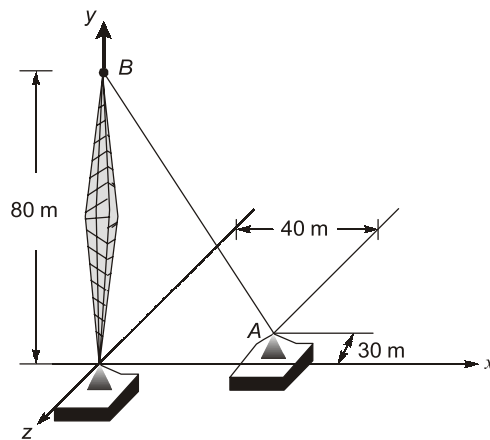
$$\therefore N_D = \sqrt{F_{HD}^2 + F_{VD}^2} = \sqrt{(-200)^2 + (600)^2} = 632.45 \text{ N}$$

Q.9 A tower guy wire is anchored by means of a bolt at A. If the magnitude of the tension in the wire is 2500 N, determine the components of the force acting on the bolt in the x, y and z directions.



[CSE (Mains) 2020 : 10 Marks]

Solution:



Simply find the unit vector, $\hat{AB} = \frac{\vec{AB}}{|\vec{AB}|}$

Coordinate of A (40, 0, -30)

Coordinate of B (0, 80, 0)

$$\vec{AB} = (0 - 40)\hat{i} + (80 - 0)\hat{j} + (0 + 30)\hat{k} = -40\hat{i} + 80\hat{j} + 30\hat{k}$$

$$|\overline{AB}| = \sqrt{(-40)^2 + (80)^2 + (30)^2} = 10\sqrt{89}$$

$$\widehat{AB} = -0.424\hat{i} + 0.848\hat{j} + 0.318\hat{k}$$

Force vector acting at the bolt A, $\vec{F} = |\overline{T_{AB}}|\widehat{AB} = 2500(-0.424\hat{i} + 0.848\hat{j} + 0.318\hat{k})$

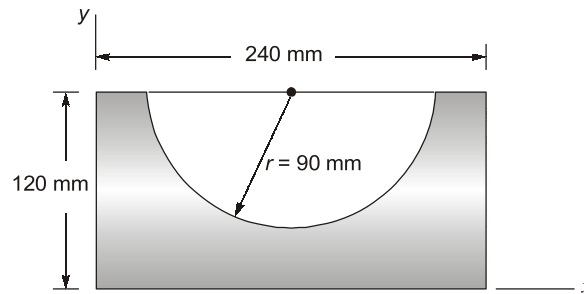
$$\vec{F} = -1060\hat{i} + 2120\hat{j} + 795\hat{k}$$

$$\vec{F}_x = -1060\hat{i} \text{ (N)}; \vec{F}_y = 2120\hat{j} \text{ (N)}$$

⇒

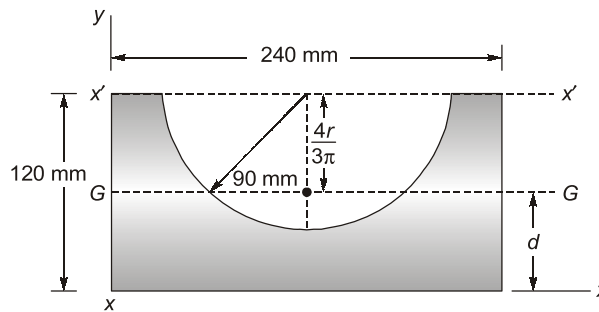
$$\vec{F}_z = 795\hat{k} \text{ (N)}$$

Q.10 Determine the moment of inertia of the shaded area with respect to the x-axis.



[CSE (Mains) 2020 : 10 Marks]

Solution:



$$[I_{\text{shaded area}}]_{xx} = [I_{\text{rectangle}}]_{xx} - [I_{\text{semicircular area}}]_{xx}$$

$$(I_{\text{rect.}})_{xx} = I_{GG} + A \times \left(\frac{h}{2}\right)^2 = \frac{bh^3}{12} + bh \times \left(\frac{h^2}{4}\right) = \frac{1}{3}bh^3$$

$$(I_{\text{rect.}})_{xx} = \frac{1}{3} \times 240 \times 120^3 = 1.3824 \times 10^8 \text{ mm}^4$$

$$[I_{\text{semicircular area}}]_{x'x'} = \frac{\pi r^4}{8}$$

$$[I_{\text{semicircular area}}]_{x'x'} = [I_{s-c}]_{GG} + A_{s-c} \times \left(\frac{4r}{3\pi}\right)^2 \quad [s-c = \text{Semicircular}]$$

$$\frac{\pi r^4}{8} = [I_{s-c}]_{GG} + \frac{\pi r^2}{2} \times \frac{16r^2}{9\pi^2}$$

$$[I_{s-c}]_{GG} = \frac{\pi r^4}{8} - \frac{16r^4}{18\pi} = r^4 \left[\frac{\pi}{8} - \frac{16}{18\pi} \right]$$

$$= 90^4 \times \left[\frac{\pi}{8} - \frac{16}{18\pi} \right] = 7.2011 \times 10^6 \text{ mm}^4$$

$$[I_{S-C}]_{xx} = [I_{S-C}]_{GG} + A_{S-C} \times d^2$$

$$d = 120 - \left(\frac{4r}{3\pi} \right) = 120 - \frac{4 \times 90}{3\pi} = 81.8028 \text{ mm}$$

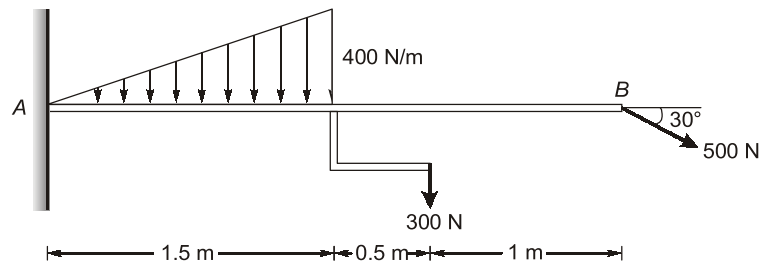
$$A_{S-C} \times d^2 = \frac{\pi \times 90^2}{2} \times 81.8028^2 = 8.5141 \times 10^7 \text{ mm}^4$$

$$[I_{S-C}]_{xx} = [7.2011 \times 10^6 + 8.5141 \times 10^7] \text{ (mm}^4\text{)}$$

$$= 9.2342 \times 10^7 \text{ mm}^4$$

$$[I_{\text{Shaded area}}]_{xx} = 1.3824 \times 10^8 - 9.2342 \times 10^7 = 4.5898 \times 10^7 \text{ mm}^4$$

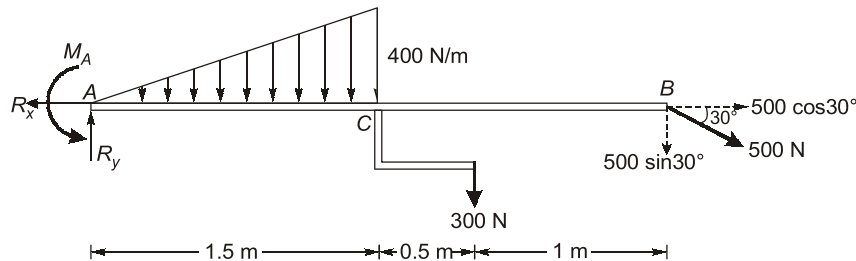
Q.11 What is the supporting force system at A for the cantilever beam shown in the figure? Neglect the weight of the beam.



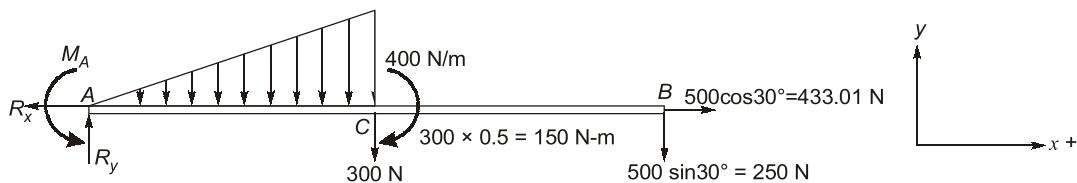
[CSE (Mains) 2021 : 10 Marks]

Solution:

Let R_x , R_y and M_A be reaction forces and moment at support A i.e.



Resolving forces along x and y axis.



As beam in equilibrium thus, $\Sigma F_x = 0$, $\Sigma F_y = 0$, $\Sigma M_{\text{about A}} = 0$

$$\Sigma F_x = 0$$

$$\Rightarrow -R_x + 433.01 = 0$$

$$\Rightarrow R_x = 433.01 \text{ N (Towards left)}$$

$$\Sigma F_y = 0$$

$$\Rightarrow R_y - \frac{1}{2} \times 400 \times 1.5 - 300 - 250 = 0$$

$$\Rightarrow R_y = 850 \text{ N (Upwards)}$$

$$\Sigma M_{\text{about A}} = 0$$

$$\Rightarrow -M_A - \left(\frac{1}{2} \times 400 \times 1.5 \right) \times 1 + 300 \times 1.5 + 150 + 250 \times 3 = 0$$

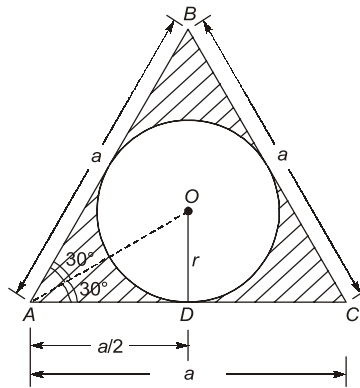
$$\Rightarrow M_A = 1650 \text{ N-m (Anticlockwise)}$$

Q.12 An inscribed circular hole is made in a triangular lamina with each side 'a'. Find the area moment of inertia of this lamina about one of the sides.

[CSE (Mains) 2021 : 15 Marks]

Solution:

Inscribed circular hole in triangular lamina of side 'a'



$$\text{In } \triangle AOD, \quad \tan 30^\circ = \frac{r}{a/2} \Rightarrow r = \frac{a}{2} \tan 30^\circ = \frac{a}{2\sqrt{3}}$$

Area moment of inertia of shaded portion about AB = (Area moment of inertia of $\triangle ABC$ about AB) - (Area moment of inertia of circular area about AB)

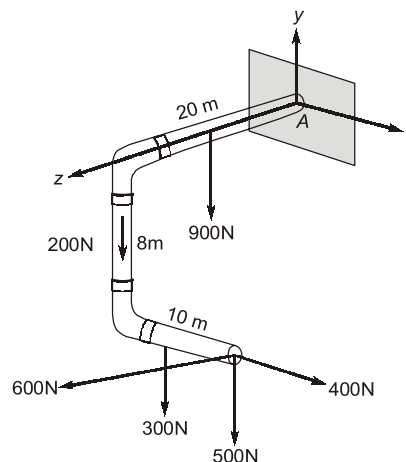
$$\begin{aligned} \text{Area moment of inertia of } \triangle ABC \text{ about AB} &= \frac{bh^3}{12} = \frac{a}{12} \times \left(\frac{\sqrt{3}}{2}a\right)^3 \quad [b = \text{base} = a, h = \text{height} = a \sin 60^\circ = \frac{\sqrt{3}}{2}a] \\ &= \frac{\sqrt{3}}{32} a^4 = 0.05412 a^4 \end{aligned}$$

$$\begin{aligned} \text{Area moment of inertia of circular area about AB} &= I_{NA} + A \cdot r^2 = \frac{\pi}{4} r^4 + \pi r^2 \cdot r^2 = \frac{5}{4} \pi r^4 \\ &= \frac{5}{4} \pi \times \left(\frac{a}{2\sqrt{3}}\right)^4 = 0.02727 a^4 \end{aligned}$$

$$\begin{aligned} \therefore I_{AB} &= 0.05412 a^4 - 0.02727 a^4 \\ I_{AB} &= 0.02685 a^4 \end{aligned}$$

Hence, the area moment of inertia of inscribed circular hole in a triangular lamina of each side a is $0.02 a^4$.

Q.13 A pipe with external forces is shown in figure. The loads 300 N, 200 N and 900 N are acting at the centres of pipe sections as shown in the figure. Find the resultant of force system at point A shown in the figure.



[CSE (Mains) 2022 : 10 Marks]

Solution:

For the given figure:

$$F_x = 400 \text{ N}$$

$$F_y = -(900 + 300 + 200 + 500) = -1900 \text{ N}$$

$$F_z = 600 \text{ N}$$

According to given directions, $F_A = (400\hat{i} - 1900\hat{j} + 600\hat{k}) \text{ N}$

Resultant moment of the system at A would be

$$\vec{M} = \vec{r} \times \vec{F}$$

∴ Moment of force acting at P :

$$M_1 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 10 & -8 & 20 \\ 400 & -500 & 600 \end{vmatrix} = \hat{i}(5200) + \hat{j}(2000) - \hat{k}(1800) \quad \dots(i)$$

Moment of force acting at Q :

$$M_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 5 & -8 & 20 \\ 0 & -300 & 0 \end{vmatrix} = 300(20\hat{i} - 5\hat{k}) = 6000\hat{i} + 1500\hat{k} \quad \dots(ii)$$

Moment of force acting at R :

$$M_3 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & -4 & 20 \\ 0 & -200 & 0 \end{vmatrix} = 200 \times (20\hat{i}) = 4000\hat{i} \quad \dots(iii)$$

Moment of force acting at S :

$$M_4 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0 & 10 \\ 0 & -900 & 0 \end{vmatrix} = 10 \times (900\hat{i}) = 9000\hat{i} \quad \dots(iv)$$

Net moment of all the forces:

$$\begin{aligned} M_T &= M_1 + M_2 + M_3 + M_4 \\ &= \hat{i}(5200 + 6000 + 4000 + 9000) + \hat{j}(2000) - \hat{k}(1800 + 1500) \\ &= 24200\hat{i} + 2000\hat{j} - 3300\hat{k} \end{aligned}$$

∴ The moment reaction at A would be

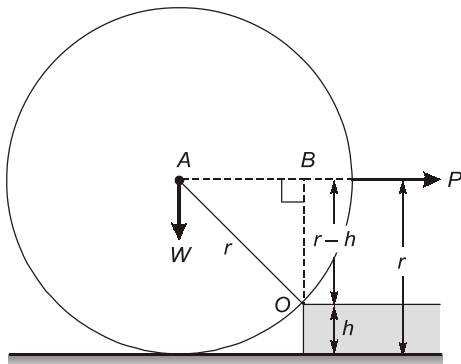
$$M_A = -24200\hat{i} - 2000\hat{j} + 3300\hat{k} \text{ Nm}$$

Q.14 A heavy carriage wheel of weight W , and radius r is to be dragged over an obstacle of height h by a horizontal force P applied to the centre of the wheel. Show that P is slightly greater than $\frac{W \cdot \sqrt{2rh - h^2}}{r - h}$.

[CSE (Mains) 2023 : 10 Marks]

Solution:

The carriage wheel with the obstacle is as shown below. The point O is the point of contact of wheel and the obstacle just before it is about to be dragged over it



For the wheel to be dragged over the obstacle, the torque due to P about O should be greater than or equal to the torque due to weight W of the wheel about point O .

Hence,

$$P \times OB > W \times AB$$

$$P > \frac{W \times AB}{OB} \quad \dots (i)$$

Now, from geometry, in ΔABO ,

$$AO^2 = AB^2 + OB^2$$

$$r^2 = AB^2 + (r-h)^2$$

$$AB^2 = r^2 - (r^2 + h^2 - 2rh) = 2rh - h^2$$

$$AB = \sqrt{2rh - h^2} \quad \dots (ii)$$

From equations (i) and (ii),

$$P > \frac{W \times \sqrt{2rh - h^2}}{r - h}$$

Hence, the horizontal force should be slightly greater than $\frac{W \times \sqrt{2rh - h^2}}{r - h}$ to cause the wheel to be dragged over.

2. Friction

Q.1 A stuntman drives a motorcycle around a circular vertical wall 30 m in diameter. The coefficient of friction between the tyre and wall is 0.6. Determine the minimum speed that will prevent sliding down the wall. Determine the angle also by which the motorcycle is inclined to the horizontal.

[CSE (Mains) 2012 : 12 Marks]

Solution:

Given $d = 30$ m, Coefficient of friction, $\mu = 0.6$

Let the minimum speed that will prevent sliding down the wall be v .

$$\text{Normal force, } N = \frac{mv^2}{r}$$

$$\text{Friction force, } f_s = \frac{\mu mv^2}{r} = \mu N$$

For equilibrium, Friction force (f_s) = Weight

$$\frac{\mu mv^2}{r} = mg$$

$$v = \sqrt{\frac{gr}{\mu}} = \sqrt{\frac{10 \times 15}{0.6}}$$

$$v_{\min} = 15.82 \text{ m/s}$$

